Optimum Sensor Locations on a Planar Surface for Load Function Characterization

Cameron W. Coates
Engineering Studies Program,
Armstrong Atlantic State University
Savannah, GA 31419 USA
cameron.coates@armstrong.edu

Darryl W. Daniell
Dept. of Computer Science
Armstrong Atlantic State University
Savannah, GA 31419 USA
darrylda@me.com

Abstract *- A Neural Network algorithm is developed in order to predict sensor locations that will recover coefficients of 2-D polynomial static continuous load functions applied transversely to a sandwich plate. Initially, strain values obtained from a Finite Element Model (FEM) are used to simulate sensor readings which serve as input for an inverse computational method which enables coefficient recovery. The Back-Propagation Neural Network (BPNN) is trained using several polynomial load cases and randomized sensor positions in order to predict optimum locations for strain measurement along a planar surface such that the load function coefficients will be accurately recovered via the inverse method. The BPNN was able to predict 2-D sensor locations such that predicted polynomial load function coefficients were within 1% of their actual value. Locations determined by the BPNN's allow predictions of coefficient values with significantly smaller fluctuations and on average better accuracy than randomly chosen locations.*

Keywords: Finite element, strain, Neural Network, inverse, sandwich plate.

1 Introduction

With increasing necessity for autonomous aircraft in military and civilian applications, it is imperative that these lifting bodies are able to detect and mathematically model the flight load distribution along their surfaces. Accurate aerodynamic load detection and modeling may also be useful for civil structures subjected to aerodynamic loading large enough to affect mechanical performance, such as bridges, high rise buildings and offshore platforms. This work assumes a scenario in which sensors can be placed on or embedded in civil structures or lifting bodies, such that real time data can be obtained as loads occur in service. Since limited or discrete sensor locations are used to model a continuous system (load function), the problem becomes an inverse one. For computational efficiency, it would be prudent to minimize the data needed to model the load functions and also arrange any sensors needed such that the most accurate estimate of the flight load function can be obtained. The choice of sensor location now becomes of utmost importance since the results will be used in an inverse method which is already approximate.

There are a variety of mathematical approaches that have been developed in order to solve inverse problems in elasticity [1-4]. Tessler and Spangler [5,6] developed an Inverse Finite Element Method (I-FEM) that provided full-field reconstruction of plate and shell deformations obtained from experimentally measured surface strains. The formulation was based upon the minimization of a least squares functional that uses the complete set of strain measures from first-order shear-deformation shell theory. Shkarayev8 and co-workers demonstrated load recovery for specific static load cases on a flat plate using this inverse method. Coates and Thamburaj [7-8] investigated this method's efficacy if load distribution functions of one and two variables were expressed as the sum of their Fourier cosine series terms. In that study, the strains obtained from the FE method were used to simulate experimentally measured strains. It was shown that if a database containing the Fourier coefficients of historical load functions exists, the most probable load distribution function could be identified in real time using discrete strain measurements. If the static load function can be accurately recovered, the question of where to position strain sensors in order to most accurately recover this function then arises. The sensor placement problem has been approached by several authors using manual optimization and intuitive placement recipes [9-14], or combinatorial optimization techniques [15-19]. A survey of optimization strategies performed by Padula et al [20] concludes that tabu searches and genetic algorithms have been effective in solving these problems. The dominant neural network algorithms used in structural applications rely on the minimization of a least squares functional for the network to learn which weights to assign to which neural connections for what input. The inverse computational method developed by Tessler and Sprangler also utilizes the minimization of a least squares functional, therefore its results may lend themselves well to the application of a neural network for the prediction of effective sensor locations for application of the inverse computational method itself.

This work describes the Back-Propagation Neural Networks (BPNN's) developed in order to predict the sensor locations that will yield consistent and accurate predictions by the inverse computational method for load function coefficients in 1-D and 2-D cases.

1.1 Inverse Method

Below is a brief explanation of the inverse method, for more details the reader may look at references [7-9]. From a set of m possible load cases on a surface, the i_{th} load approximation may be expressed in parametric form;

$$F_i(s) = \sum_{j=1}^{l} a_{ij} R_{ij}(s) \tag{1}$$

where the a_{ij}'s are unknown approximation parameters and the $R_{ij}(s)$ are spatial distribution functions. By using Eqn. (1) in the Finite Element Method, the displacements, strains, and stresses, corresponding to the i_{th} load may be computed from

$$\{x_i\} = \sum_{j=1}^{l} a_{ij}\{x_{ij}\} \tag{2}$$

where x_i represents any of these quantities.

If $\boldsymbol{\varepsilon}_i^*$ represents the measured strains for the i_{th} load case and $\boldsymbol{\varepsilon}_i$ represents the strains measured due to each R_{ij} load individually applied, the coefficients in Eqn. (1) may then be computed by performing a least squares minimization of S_i in Eqn. (3) below, with respect to the parameter a_{ij}, where

$$S_i = \{\boldsymbol{\varepsilon}_i - \boldsymbol{\varepsilon}_i^*\}^T \{\boldsymbol{\varepsilon}_i - \boldsymbol{\varepsilon}_i^*\}$$
$$= \left\{\sum_{j=1}^{m} a_{ij}\{\boldsymbol{\varepsilon}_{ij}\} - \boldsymbol{\varepsilon}_i^*\right\}^T \left\{\sum_{j=1}^{m} a_{ij}\{\boldsymbol{\varepsilon}_{ij}\} - \boldsymbol{\varepsilon}_i^*\right\} \tag{3}$$

This minimization can be also performed while accounting for certain inequity constraints, for example

$$\varphi(a_{ij}) \geq 0 \tag{4}$$

The a_{ij} coefficients are then obtained by solving the resulting system of linear algebraic equations. In a practical scenario, the function will not be known *a priori*; Ref. [7] suggests that Chebyshev polynomials may be chosen to represent the R_{ij} spatial functions. A quality function based on computer simulations and experimental statistics could then be employed in order to select the most appropriate load case. Previous work by the authors of this paper proposed expressing the unknown function in Eqn. (1) in terms of its single or double Fourier cosine series.

Ref. [7] also introduced two loading cases of the general form, $p(x,y)=b_1x^2+b_2y^2+b_3y+b_4$; a dominant bending load given by b_1=-0.25, b_2=-4, b_3=0 and b_4=1.5 and a dominant torsional load where b_1=-0.25, b_2=-4, b_3=15 and b_4=0.67, which were applied perpendicular to the surface at the top of the plate. The R_{ij}'s in this case are the loads x^2, y^2, y and 1. The $\{\boldsymbol{\varepsilon}_i\}$'s obtained from each of these R_{ij} load cases along with the $\{\boldsymbol{\varepsilon}_I^*\}$'s obtained from the distributed load $p(x,y)$ were then used in Eqn. (3). Assuming a load distribution of $F(x,y)= a_1x^2+a_2y^2+a_3y+a_4$ and using the inverse approach, the a_j (j=1….4) coefficients compared favorably with the actual load case as well as values obtained in Ref. [7].

2 Finite Element Model

An initial Finite Element model of a cantilever sandwich plate was developed using commercial FE software. Loading was applied transversely as shown in Fig 1. The model was discretized with 2902 second-order solid parabolic tetrahedral elements with 4905 nodes, each node having three degrees of freedom. The Young's Modulus, E_o, and Poisson's ratio, v_o, for the outer plates were 73 GPa and 0.33 respectively. For the inner core, the Young's Modulus E_i, and Poisson's ratio v_i, were 6.1 MPa and 0.49 respectively. The dimensions of the inner core as well as the outer plates were 5m x 3m x 0.3m.

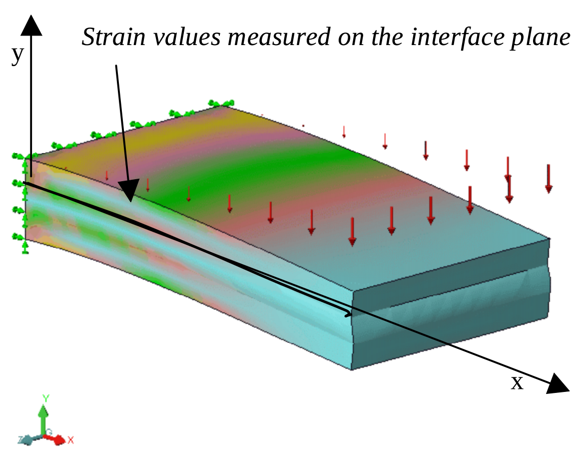

Figure 1. FE model of Transversely Loaded Cantilever Sandwich Structure.

3 Neural Network for 2-D

A Back-Propagation Neural Network was generated in order to predict the five locations along the interface where strain sensors could be placed in order to most accurately recover the coefficients via the inverse method. The network consisted of an input layer, two hidden layers and an output layer. The BPNN was designed to accept six inputs, which include the coefficients of x^2, y^2, x^1, the sandwich plate aspect ratio (height vs width), the core thickness and the ratio of the Young's modulus of plates to the Young's modulus of the core. The second and third layer had five neurons and used the logsig transfer function shown in Equation 5. The fourth layer had 10 neurons (output) and used the purelin transfer function sown in Equation 6. The ten output neurons represented the x and y coordinates of the strain measurement locations along the upper interfacial plane.

$$f(n) = \frac{1}{1+e^{-n}} \quad (5)$$

The number of nodes in the hidden layers was determined using trial and error. The Levenberg-Marquardt algorithm was used in training the network due to its relatively low demand on computer memory (compared to Newton's method) and a history of successful use in similar applications [21].

The BPNN weights were updated using the batch gradient descent with momentum method. Thirty one sets of (five) locations were chosen using a pseudorandom number generator code. For each set of five locations, the five pairs of strain (ε_{xx} and ε_{yy}) resulting from the applied load were obtained. These values were then used to recover the load function coefficients. The sixty-two coefficients (31 pairs) obtained from the inverse method, along with the transformed flexural rigidity of the sandwich plate, D, where

$$D = \frac{E_o t h^2}{2\left(1-\nu_o^2\right)^2} \quad (6)$$

(h = core thickness, t = outer plate thickness), and a polynomial order factor based on the applied load function, provided input data for training the BPNN. The 2-D load functions used were $2x^2 + 10y^2$ and $2x^2 + 30y^2 + 10x$, where x and y represent the axial and transverse distances, respectively, in the horizontal plane measured from the cantilevered end.

4 Results

The five planar locations (coordinate pairs) for measuring strain values predicted by the BPNN when the load function $f(x) = 2x^2 + 10y^2$ was applied yielded coefficients that differed from the actual values of 2 and 10 by 0.067% and 0.67%, respectively. These values were well within the average deviations of the coefficient data set from the actual values, which were 0.08% and 0.75% respectively. Figures 2 and 3 illustrate this coefficient data set in a scatter plot, the relative locations for the average distance and the network predicted coefficient. Similar analysis of the load function $f(x) = 2x^2 + 30y^2 + 10x$ yielded coefficients that differed from their actual values (i.e. 2, 30, and 10) by 2%, 3.33% and 2% respectively. Again, these results were well within the average deviations of all individual predictions of 5.5%, 18.3% and 10% (obtained from strains measured at randomly chosen locations).

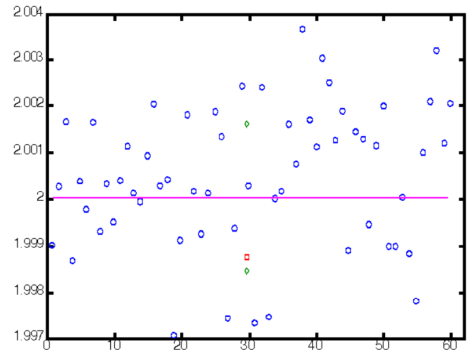

Figure 2. Comparison of x^2 coefficient predicted by inverse method using BPNN output locations to x^2 coefficient predictions by inverse method using randomly chosen locations for the quadratic load function $f(x, y) = 2x^2 + 10y^2$

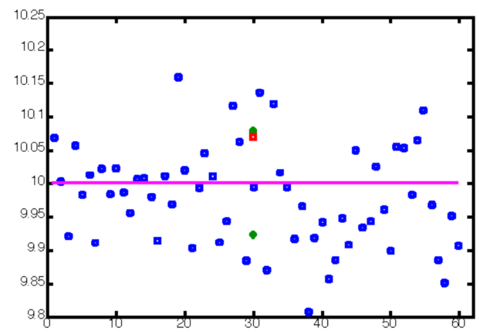

Figure 3. Comparison of y^2 coefficient predicted by inverse method using BPNN output locations to y^2 coefficient predictions by inverse method using randomly chosen locations for the quadratic load function $f(x, y) = 2x^2 + 10y^2$

5 Conclusions

An inverse computational method using strains derived from FEA has been successfully applied to recovering the coefficients of 2-D load functions applied transversely to a sandwich plate with isotropic outer plates and core. The inverse method assumes that *a priori* knowledge of the polynomial form of the function exists. This could be obtained from a historical database, experimental testing, statistical or theoretical methods or a combination of these methods.

Back-Propagation Neural Networks have been developed that select the locations for strain measurements such that the coefficients of the load function are recovered with reasonable accuracy. For all functions chosen, the coefficient values recovered from the BPNN have been closer to the actual value than the average deviation of the coefficient predictions from the individual randomly chosen locations used to train the BPNN. Successful results were demonstrated with polynomial 2nd order functions of two variables for the 2-D case. However, the FE models used ignore shear effects at the interface and assume a perfect bond at the interface, therefore the sandwich plate. This work therefore forms the basis for development of a more comprehensive model where the optimum locations may be predicted for strain sensors such that the inverse method will accurately recover nonlinear functions for sandwich plates where anisotropy, interface shear stresses and shear stress distributions are considered.

Acknowledgment

The authors are grateful to the NASA Space Grant Consortium for supporting this work by funding undergraduate research at Armstrong Atlantic State University.

References

[1] A.M. Maniatty and N.J. Zabaras, “Investigation of regularization parameters and error estimating in inverse elasticity problems,” *International Journal of Numerical Methods in Engineering*, Vol. 37, No. 6, pp.1039-1052, 1994.

[2] A. Tarantola, *Inverse Problem Theory Methods for Data Fitting and Model Parameters Estimation*, Elsevier, New York, USA, 1987.

[3] B.P. Bogert, E. Haugse, and R.E. Gehrki, “Structural shape identification from experimental strains using a modal transformation technique,” Proceedings of the 44th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference, Norfolk, VA, pp. 2003-1626 April 7-10, 2003.

[4] G.C. Kirby, T.W. Lim, R. Weber, A.B. Bosse, C. Povich and S. Fisher. “Strain based shape estimation algorithms for a cantilever beam,” Proceedings of the Intl. Society for Opt. Engr. Conference, San Diego, CA, pp. 788-798, March 3-6, 1997.

[5] A. Tessler and J. Spangler, “A least-squares variational method for full-field reconstruction of elastic deformations in shear-deformable plates and shells,” *Computer Methods in Applied Mechanics and Engineering* Vol. 194, No. (2-5), pp. 327-339, 2005.

[6] S. Shkarayev, R. Krashantisa and A. Tessler, “An inverse interpolation method utilizing in-flight strain measurements for determining loads and structural response of aerospace vehicles,” 3rd International Workshop on Structural Health Monitoring, Stanford, CA, 2001. NASA Tech. Rep. 31WSHM.

[7] C.W. Coates, P. Thamburaj and S. Scott, “Double Fourier applications for real time aerodynamic load recovery,” Proceedings of the 62nd Annual American Helicopter Society (AHS) Forum, Phoenix, AZ, pp. 1322-1329, May 9-11 2006.

[8] C.W. Coates, P. Thamburaj and C. Kim, “An inverse method for selection of Fourier coefficients for flight load identification,” Proceedings of the 46th Annual AIAA/ ASME/ASCE/AHS Structures, Structural Dynamics and Mechanics Conference, Austin, TX, pp. 1-7, April 18-21, 2005.

[9] X. Cao, Y. Sugiyama and Y. Mitsui, “Application of artificial neural networks to load identification,” *Computers and Structures*, Vol. 69, No. 1, pp. 63-78, 1998.

[10] S. Kwon, H. Lee and S. Shin. “Optimal accelerometer locations for structural health monitoring using time-domain system identification,” *Key Engineering Materials*, Vol. 321-323, pp. 273-277, 2006.

[11] M. Sunar and S.S. Rao, “Distributed modeling and actuator location for piezoelectric control systems,” *AIAA Journal*, Vol. 34, No. 10, pp. 2209-2211, 1996.

[12] C. Liang, F.P. Sun and C.A. Rogers, “Determination of design of optimal actuator location and configuration based on actuator power factor,” *J. of Intelligent Material Systems and Structures*, Vol. 8, No. 4, pp. 344-352, 1997.

[13] A. Bhargava, Z. Chaudhry, C. Liang and C.A. Rogers, “Experimental verification of optimal actuator location and configuration based on actuator power factor,” *J. of Intelligent Material Systems and Structures*, Vol. 6, No. 3, pp. 411-418. 1995.

[14] Y.K. Kang, H.C. Park, W. Hwang and K.S. Han, “Optimum placement of piezoelectric sensor/actuator for vibration control of laminated beams,” *AIAA Journal*, Vol. 34, No. 9, pp. 1921-1926, 1996.

[15] A. Hac and L. Liu, “Sensor and actuator location in motion control of flexible structures,” *J. of Sound and Vibration*, Vol. 167, No. 2, pp. 239-261, 1993.

[16] M. Papadopoulos and E. Garcia, “Sensor placement methodologies for dynamic testing,” *AIAA Journal*, Vol. 36, No. 2, pp. 256-263, 1998.

[17] M.T. Simpson and C.H. Hansen, “Use of genetic algorithms to optimize vibration actuator placement for active control of harmonic interior noise in a cylinder with floor structure,” *Noise Control Engr. J.*, Vol. 44, No. 4, pp. 169-184, 1996.

[18] W. Gawronski, "Actuator and sensor placement for structural testing and control," *J. of Sound and Vibration*, Vol. 208, No. 1, pp. 101-109, 1997

[19] D.C. Kammer and L. Yao, "Enhancement of on-orbit modal identification of large space structures through sensor placement," *J. of Sound and Vibration*, Vol. 171, No. 4, pp. 119-139, 1994.

[20] S. Pottie and D. Botteldooren, "Optimal placement of secondary sources for active noise control using a genetic algorithm," Proceedings of Internoise 96 25th Anniv. Congress, Liverpool, England, pp. 1101-1104, 1996.

[21] H. Demuth and M. Beale, *Neural Network Toolbox for Use with MATLAB, Users Guide*, Version 4, The Math Works, Natic, MA, 2000.